Written Exam Economics summer 2016

Industrial Organization

August 11, 2016

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam paper consists of three pages in total, including this one

Attempt both questions.
Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

Question 1: Dynamic monopoly and the Coase conjecture (the two-period "movie model")

This is a model of a monopoly firm that sells its good in two time periods to consumers who are forward-looking. It is identical to a model that we studied in the course, and it is related to the so-called Coase conjecture.

There are two time periods, $t = 1$ and $t = 2$. At each t, a monopoly firm is producing and selling a good. There are a continuum of consumers who differ from each other with respect to the parameter $r \in [0, 1]$, the gross utility from consuming one unit of the good during one time period. The r's are uniformly distributed on $[0, 1]$. A consumer gets utility from consuming (a single unit of) the good only *once*, and therefore never wants to consume the good in both periods. A consumer's net utility from consuming the good in period t equals $r - p_t$, where p_t is the price the firm charges in period t. Not buying the good yields the utility zero. The consumers' (common) discount factor is denoted by $\delta \in [0, 1)$. The timing of events is as follows.

1. The monopoly firm chooses its first-period price p_1.
2. The consumers observe p_1 and then (simultaneously) choose whether to buy or not.
3. The monopoly firm chooses its second-period price p_2.
4. The consumers observe p_2 and then (simultaneously) choose whether to buy or not.

The monopoly firm has a constant marginal cost of production, which is normalized to zero. The objective of the firm is to maximize its profits; however, the firm's discount factor equals zero, which means that when choosing p_1 at stage 1, its second-period profit has zero weight. The consumers maximize their net utilities, appropriately discounted.

(a) Solve for a subgame perfect Nash equilibrium of the model in which consumers with $r > \widehat{r}$, for some $\widehat{r} \in (0, 1)$, consume in period 1. Find the equilibrium value of $\widehat{r}$. Also identify the equilibrium values of p_1 and p_2.

(b) Explain in words what the Coase conjecture says. Also explain the intuition.

(c) Define the "Herfindahl index" and the "3-firm concentration ratio". Also, consider a market with seven firms. Their market shares are 5, 5, 10, 10, 20, 20 and 30 percent. Calculate the Herfindahl index and the 3-firm concentration ratio for this market.

Question 2: Collusion with fluctuating and persistent demand

Consider the following version of the Rotemberg-Saloner model. In a market there are two ex ante identical firms, indexed by $i \in \{1, 2\}$. They produce a homogeneous good and each firm has a constant marginal cost $c \geq 0$. There are infinitely many, discrete time periods t (so $t = 1, 2, 3, \ldots$), and at each t the firms simultaneously choose their respective price, p_i^t. The firms' common discount factor is denoted by $\delta \in [0, 1)$. As the good is homogeneous, demand is a function of the lowest price, $p^t = \min\{p_1^t, p_2^t\}$. If the firms charge the same price, then they share the demand equally.

Demand in each period t is either high ($q^t = D_H(p^t)$) or low ($q^t = D_L(p^t)$), with $D_H(p^t) >$

$D_L(p^t)$ for all p^t. Demand realizations are *not* necessarily independent across time. Instead, the demand state follows a so-called Markov chain. That is, the probability that a, say, high state realizes in period $t+1$ depends on the state in period t. In particular, it is assumed that:

$$\Pr\left[\text{high demand in } t+1 \mid \text{high demand in } t\right] = \alpha,$$

$$\Pr[\text{high demand in } t+1 \mid \text{low demand in } t] = \beta,$$

where $\alpha \in (0,1)$ and $\beta \in (0,1)$. In words, if the demand state is high in one period, then it stays the same in the following period with probability α. If the demand state is low in one period, then it switches (to the high state) with probability β.

The firms can observe all rival firms' choice of price once it has been made. Moreover, the firms can observe the current period's demand realization before choosing their price. However, the demand realizations in future periods are not known to the firms.

Let p_s^m be the state $s \in \{L, H\}$ monopoly price, i.e., the price that maximizes $(p-c)\,D_s(p)$. Exactly as in the course, consider a grim trigger strategy in which each firm starts out charging the price $p_s^t = p_s^m$ if the period t state is s. However, if there has been any deviation from that behavior by anyone of the firms in any previous period, then each firm plays $p_s^t = c$.

In order to investigate under what conditions the above grim trigger strategy is part of a subgame-perfect Nash equilibrium, we can use the methodology that we employed in the course when studying collusion with unobservable actions (the Green-Porter model). Thus, consider the following two equations:

$$V_H = \frac{\pi_H^m}{2} + \delta\left[\alpha V_H + (1-\alpha)\,V_L\right], \qquad (1)$$

$$V_L = \frac{\pi_L^m}{2} + \delta\left[\beta V_H + (1-\beta)\,V_L\right]. \qquad (2)$$

(a) Explain in words what equations (1) and (2) represent. Then use the notation V_H and V_L to state the two "Nash conditions" (on the equilibrium path) that are required for a firm not to want to deviate from the equilibrium — one condition for the high-demand state and one for the low-demand state.

- You should indeed state the conditions in terms of V_H and V_L. To answer this question you are not required to solve the equation system (1) and (2).

(b) If $\alpha = \beta = \frac{1}{2}$, then the model simplifies to the one that we studied in the course, where the demand states are independent across time. In that model we showed that the Nash condition for the high-demand state is more stringent than the one for the low-demand state: The incentive to deviate is strongest when demand is high. Explain (in words only) the economic reason for this result.

Now assume $\beta = 1-\alpha$, $\pi_H^m = 6$, and $\pi_L^m = 4$. In addition, let $\alpha > \frac{1}{2}$. Solving the equation system (1) and (2) for V_H and V_L yields

$$V_H = \frac{3(1-\delta\alpha) + 2\delta(1-\alpha)}{(1-\delta)[1-\delta(2\alpha-1)]}, \qquad (3)$$

$$V_L = \frac{3\delta(1-\alpha) + 2(1-\delta\alpha)}{(1-\delta)[1-\delta(2\alpha-1)]}. \qquad (4)$$

(c) In this model with $\alpha > \frac{1}{2}$, which one of the two Nash conditions is the most stringent? That is, when is collusion most difficult to sustain — in a high or in a low state?

(d) Consider again the Nash condition that is most stringent. Is this constraint relaxed or tightened if α increases? That is, does a higher value of α make collusion easier or harder?

End of Exam